


CARTESIAN PIRATES FOR TACKLING STUDENTS' MISCONCEPTIONS AND ENHANCING GRAPH SKETCHING SKILLS AMONG MATRICULATION STUDENTS

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Article Info	ABSTRACT
<p>Article history: Received: 30 Dec 2025 Revised: 4 Jan 2025 Accepted: 25 Jan 2025 Published: 1 Feb 2025</p>	<p>Graph sketching is an important aspect in Mathematics. Students are exposed to graph sketching from Form 1 and up until Form 5. However, many students who show lack of interest in carrying out graph sketching, struggle in sketching and display poor knowledge retention. Peperiksaan Semester Program Matrikulasi (PSPM) 2021/22 and 2022/23 examination reports cite students performing poorly in graph sketching. Hence to address the above issues, Cartesian Pirates are invented. Students will be allowed to explore graph sketching using a story-mode setting, discovering their misconception hence correcting and learning graphs at their own pace. Importantly, knowledge should be built as building blocks with game-setting mode, students are challenged to better themselves at graph-sketching skills. A target group of 14 students was identified and an intervention plan was devised using the Kemmis-McTaggart (1988) model. Cartesian Pirates product was utilised to create an engaging learning environment, fostering active student participation in graph sketching for these students. Data on students' performance, like scores and post-test results were collected to measure the impact of Cartesian Pirates on students' proficiency in sketching. Students' emotions and feedback were reviewed through post-intervention discussion, obtaining qualitative insights on experience. It was observed that 14 students who were given intervention using Cartesian Pirates game displayed improved scores in post-test 1 displaying mastery of 80% score of graph sketching skill from 5 types of graphs given. Upon undergoing cycle 2, students displayed better scores in post-test 2 which was 90%. Qualitative feedback also indicated that students enjoyed more, remembered steps better and could avoid misconception which happened before the intervention.</p>
<p>Keywords: Graph-sketching game learning Fun-Learning future education</p> <p></p>	

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INTRODUCTION

The teaching and learning of Mathematics at Perak Matriculation College has been drafted in line with the 2022/23 curriculum outlined by the Matriculation Department. Lessons conducted consisted of lectures and tutorial sessions. Graph sketching plays an important role in Mathematics syllabus throughout Semester 1 and 2. Many topics rely on students' knowledge of graph sketching. Failure to grasp the skill or having misconceptions would affect students' progress in the topics of Limits, Continuity and Differentiation for Semester 1. In Semester 2, it would affect chapters namely Numerical Methods, Integration and Random Variable. These topics carry marks between 16 marks to 23 marks out of 100 marks respectively. The worst-case scenario would mean, students inability to sketch the graph which constitutes a total 23 marks lost.

Realising the importance of technology in education as a future way forward, most lecturers use graph dance videos found in YouTube and sketching applications such as Desmos in teaching and learning. However, students still exhibit carelessness and have a lack of motivation in carrying out sketching. In 21st century learning, it is important for lecturers to be innovative in optimising technology to aid personal learning and improve pedagogical concepts contributing to meaningful learning. This will be crucial in improving students' performance to sketch graphs correctly and equip them with 21st century learning skills. Hence, an action research based on Kemmis and McTaggart (1988) model was conducted using Cartesian Pirates to tackle students' misconception and enhance graph sketching skills among Matriculation students.

LITERATURE REVIEW

To investigate the existence of issues faced by students of batch 2023/2024, a short test comprising questions on graphs learnt in secondary school was administered. This was done a few weeks prior to the teaching and learning of Chapter 5: Functions and Graph. First, it was to gauge their prior understanding and the misconceptions presented. Secondly, it was to tailor learning patterns to cater their needs. New knowledge should be laid out by correcting previous misconceptions (Dubinsky, 2001). The short test yielded perplexing results. From 20 objective questions given, 8 out of 14 students obtained less than a 50% score. Meanwhile, the graph sketching section showed 10 out of 14 students obtaining scores of 50% and lower. Questions were chosen from past SPM questions.

Graph sketching is a component to which students have been exposed to since Form 1. However, despite 5 years of exposure prior to Matriculation studies, it is quite underwhelming, with students showing less engagement and a lack of interest in carrying out graph sketching. It is also worth noting that despite discussing basic graphs such as quadratic function learned in Form 2, slight modifications to the function such as adding negative signs cause students to pause and think. On the next course of action, more students failed to answer questions correctly as the question got modified. Students also seem to have little knowledge retention for graph sketching. The knowledge seems to wine-out in a span of a few weeks thus putting the burden on lecturers to reteach just before *Peperiksaan Semester Program Matrikulasi (PSPM)*.

Despite efforts of using graphical apps such as Desmos which students appreciate and enjoy, the shortcomings continue to be reported by examiners every year under *Laporan Kerja Calon*. Graph sketching is required in 3 out of 6 chapters of examinable chapters; it is a chance of 50% and plays a big role in 23 marks out of 100 marks. Perak Matriculation College uses Intellectual Discussion as a platform for lecturers to discuss issues arising from teaching and learning in Mathematics. Many Mathematics lecturers at Perak Matriculation College echo the same sentiment as above while discussing strategies to improve students' knowledge on this chapter.

Table 1: Example of questions modification and no of students obtaining wrong answers

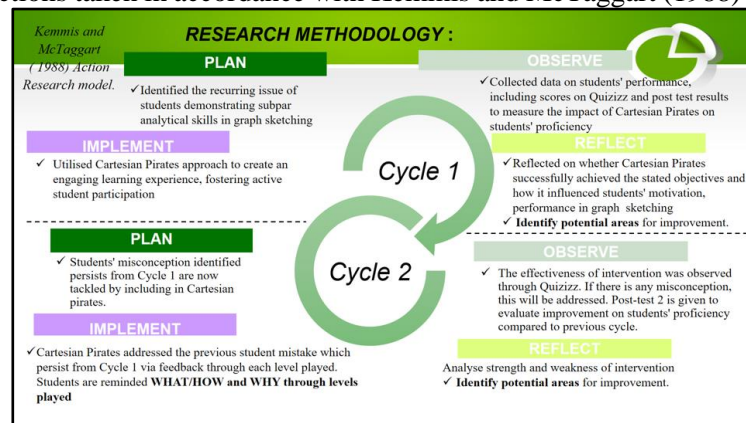
Question	No. of students who obtained wrong answer	Question	No. of students who obtained wrong answer
$y = -3x^2$	6/14	$y = x-1 $	7/14
$y = -3x^2+6$	8/14	$y = - x-1 $	10/14
$y = -3x^2-3$	8/14	$y = - x-1 + 3$	13/14

From the above discussions, it can be summarized that students feel less engaged in performing graph sketching. It must be highlighted that they have misconceptions on ‘why’ a certain shape is such and ‘how’ a particular transformation takes place. Meaningful learning must be created to ensure long-term storage of memory. Students enjoying graphing apps, *Desmos* seemed to be an eye-opener in finding solutions to students’ shortcomings. Graph sketching on handphones with some added elements is hoped to aid students learning for the long-term and most importantly to tackle misconceptions.

METHODOLOGY

By using the model by Kemmis and McTaggart (1988), the research was carried out as two-cycles. The first cycle involves students aiming to achieve 80% graph sketching proficiency under gamification using Cartesian Pirates. The second cycle would be trying to improve the students’ proficiency level by correcting remaining misconceptions to achieve a better score. The four stages involve planning, implementing, observing, and reflecting. Under the traditional approach of teaching and learning graph sketching, students demonstrated unsatisfactory results.

Figure 1: Summary of actions taken in accordance with Kemmis and McTaggart (1988) action research model



To confirm the stated issue, a pre-test was administered to a group of 14 students from class of B3T11, students of module 3: One-Year programme. 11 out of 14 students scored less than 50% indicating poor grasp of graph sketching knowledge, lack of drive to attempt and displaying misconceptions. Under lecture setting, students have already studied the 5 types of graph before the tutorial session. To gain better insight into the issue, students were asked to share their thoughts on the shortcomings. This is hoped with these insights, the stated issue could be tackled. The summary is stated as follows:

- Forgetfulness: All students who were interviewed mentioned they struggled to remember steps of sketching. 7 students cited they had problems remembering the shape of basic graphs.
- Lack of motivation: 8 students mentioned they did not feel motivated to carry out the task.
- Confusion in manipulating function: 6 students reported they were confused with how manipulation was done when the function was modified.
- Lack of support: 7 students mentioned although *Desmos* aided students to confirm the shape of the graph, support such as immediate feedback would be helpful during the learning process.

- v) Fast-paced: 8 students felt the pace in lecture was fast. Although it was slower in the tutorial, they wished the lecture pace was slower.

Self-reflection combined with qualitative insights from lecturers and students then the researcher came up with the intervention of graph sketching lessons using Cartesian Pirates.

CARTESIAN PIRATES INTERVENTION

The implementation of gamification for graph sketching using Cartesian Pirates follows an organised process as follows.

Table 3: Implementation steps of gamification for graph sketching using Cartesian Pirates

Steps	Description
Preparation	<ul style="list-style-type: none">- Students were briefed on how Cartesian Pirates would be used.- There were 5 levels consisting of 5 types of graph.- They will be aided by flash-rule cards for each stage.- Read the rule card before play just as how a board game is played.
Cartesian Pirates game	Students worked in pairs to play the game. They were required to clear each level before progressing to the next stage.
Quizizz stage	Lecturers created Quizizz based on the potential misconception of students for graph sketching. This misconception list was compiled from student errors and feedback from lecturers. Based on students' answering patterns, the lecturer discussed errors committed via Quizizz answer analysis.
Post-session feedback	Students were interviewed to share their experience and feedback on the use of Cartesian Pirates.

The game is designed such that:

- i) Students would be able to see swift and steady improvement.
- ii) Students would be able to build a perception of being good at an activity.
- iii) Students would not face difficulties in getting started.
- iv) Students would not be stuck during the activity.
- v) Lack of trust must not arise in the game.
- vi) Intrinsic boredom with the subject matter or activity itself should not happen.

Figure 2: Flashcards for each level flashcards are made up of tips and tricks for graph sketching

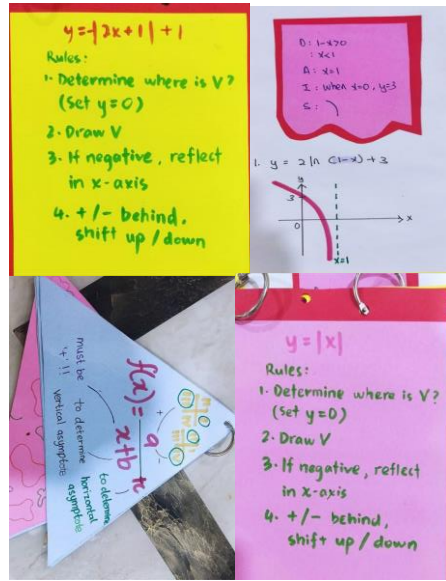


Figure 3: A screenshot of a video game covering 6 aspects mentioned in page 3 for gamification



RESEARCH FINDINGS AND DISCUSSIONS

A post-tests was conducted to assess the research objective which aims to demonstrate a minimum mastery of 80% score of graph sketching skill from 5 types of graphs given. The marking was done by an experienced lecturer from the Mathematics Unit to avoid any forms of biasness. Below is the data of pre-test (before intervention) and post-test 1 and 2 (after intervention using Cartesian Pirates).

Figure 4: Comparison of pre-test, post-test 1 and post-test 2 scores of students before and after intervention

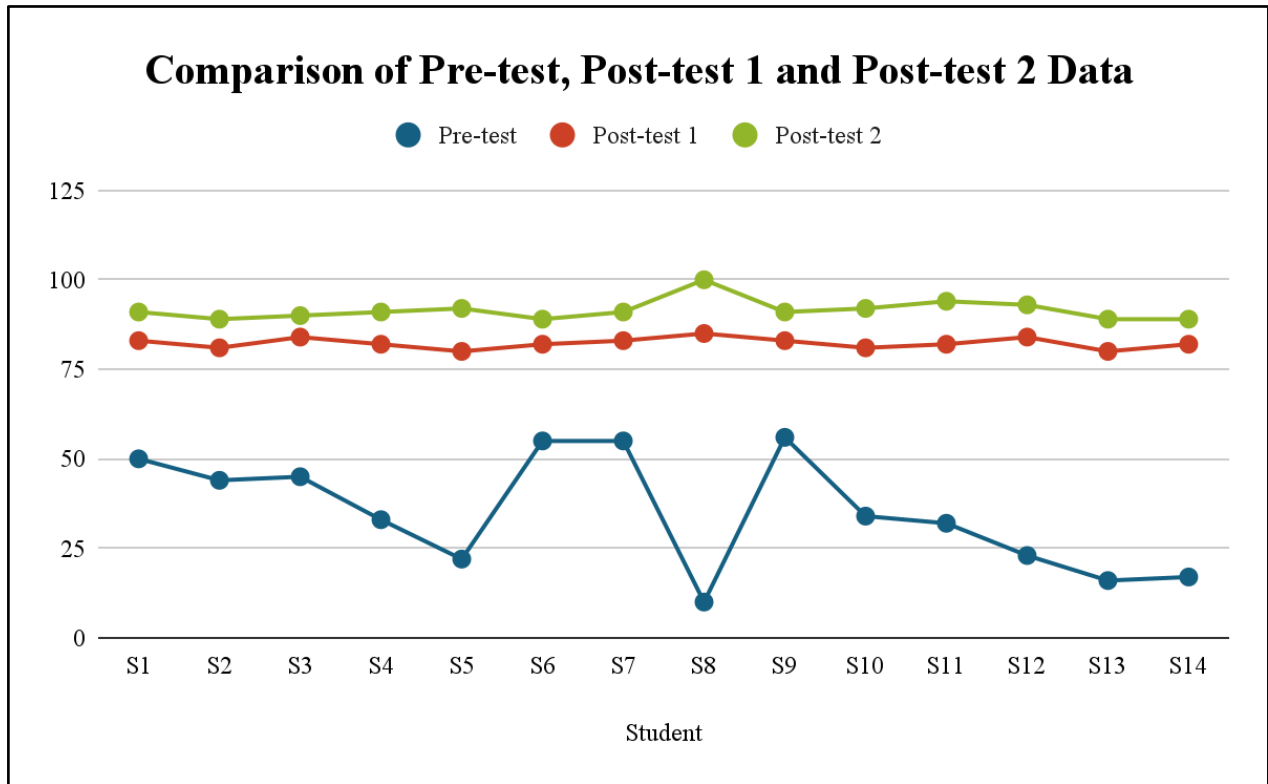


Table 4: Comparison of pre-test, post-test 1 and post-test 2 scores of students before and after intervention

No.	Pre-test (%)	Grade	Post-test 1 (%)	Grade	Post-test 2 (%)	Grade
S1	50	C	83	A	91	A
S2	44	D+	81	A	89	A
S3	45	C-	84	A	90	A
S4	33	F	82	A	91	A
S5	22	F	80	A	92	A
S6	55	C+	82	A	89	A
S7	55	C+	83	A	91	A
S8	10	F	85	A	100	A
S9	56	C+	83	A	91	A
S10	34	F	81	A	92	A
S11	32	F	82	A	94	A
S12	23	F	84	A	93	A
S13	16	F	80	A	89	A
S14	17	F	82	A	89	A
Avg	35.1		82.3		91.5	

The quantitative data obtained indicates the research objective achieved. The pre-test results did not show students attaining the minimum 80% of graph sketching proficiency and class average at 35.1%. After intervention, all 14 students attained the required skills. However, there were still students who committed

errors on certain graphs which hindered better scores. Below is the summary of errors committed by students in post-test 1 where marks were awarded for correct shape and intercept values.

Table 5: Graph and number of students committing error on shape and intercept values

Type of graph	Shape	x-intercept	y-intercept
Surd	1	0	0
Absolute value	1	0	1
Reciprocal	2	0	1
Exponential	3	2	2
Logarithmic	3	1	2

Hence, students were explained on the mistakes and errors committed. In cycle 2, they were treated to another set of Cartesian Pirates which was designed to tackle the remaining mistakes and misconceptions process described in *Figure 1*. Post-test 2 was then administered to see the effectiveness of Cartesian Pirates in improving the proficiency of students. Almost every student displayed an increase in score. Students' average rose from 82.3% to 91.5% with 10 out 14 students obtaining above 90%. Minor mistakes such as not labeling were the reason they lost marks. Qualitative data were next obtained from students to gain insights into the effect of Cartesian Pirates in reshaping students' learning experience. Below are some excerpts from the interview after the session:

Student 1: Flash cards help me to remember the steps better. I feel like it's a rule card which I must read just like before I play any board game hence abide by it.

Student 2: Since Cartesian Pirates is telling me to write/wrong in a very funny way, I try to be very careful before I answer. Hence, I think before answering since I don't want to keep repeating the same mistake.

Student 3: The immediate feedback during the use of Cartesian Pirates helps my understanding better. It also helps my confidence during the learning process.

Student 4: I focus on every tiny detail now that my lecturer always stresses in teaching. I see the connections.

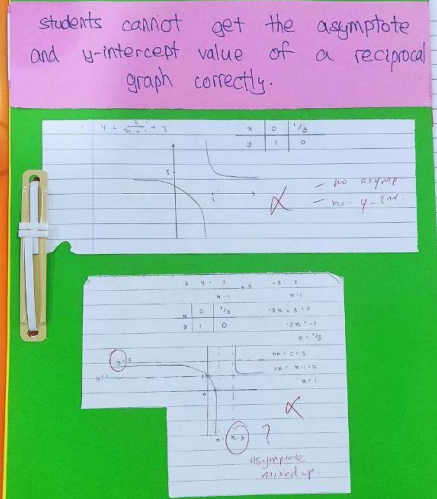
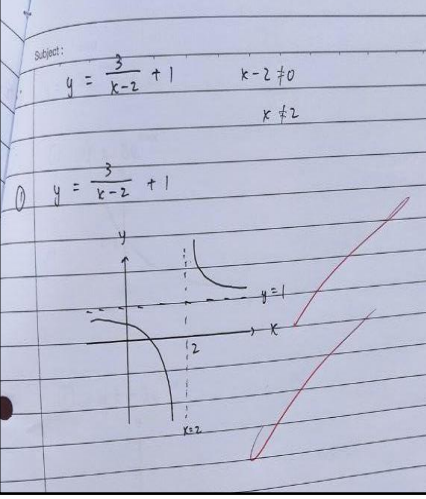
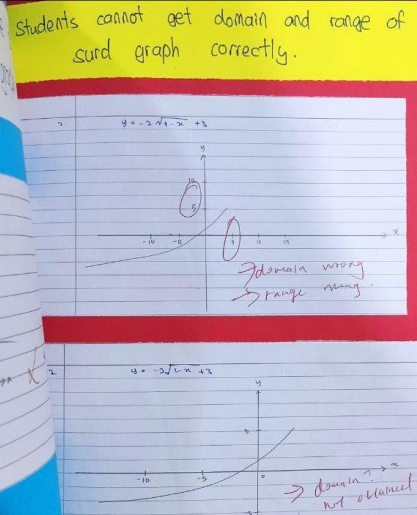
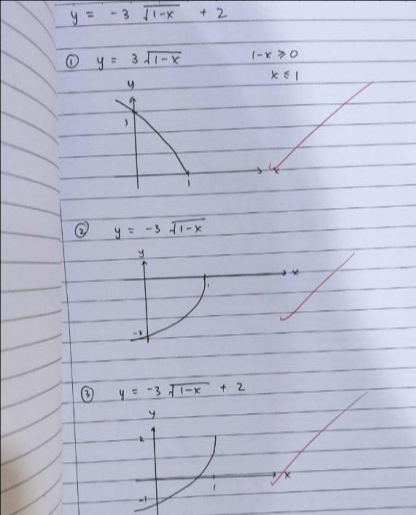
Student 5: I played Cartesian Pirates many times as I feel I can do at my own time. Then I feel excited to answer Quizizz.

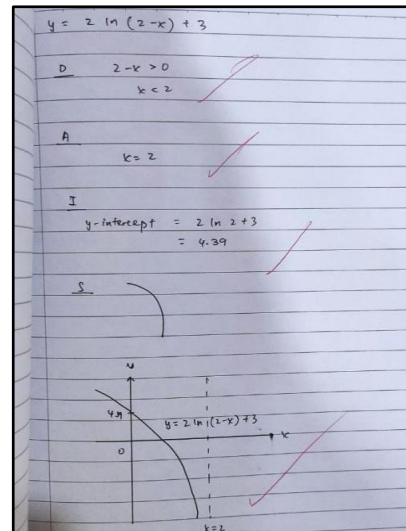
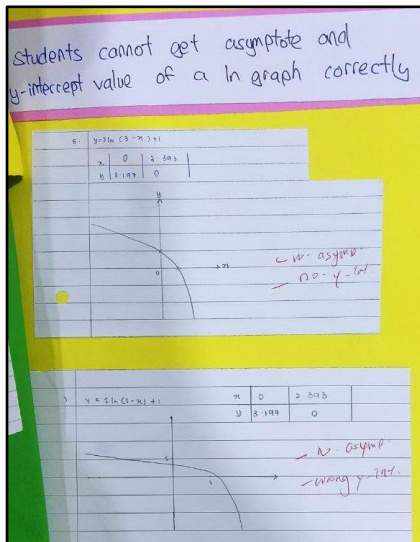
The finding above is in line with several other similar studies and an environment of effective learning. Zakaria et al. (2008) through a study on Matriculation students revealed the association between Mathematics anxiety and achievement amongst students. It is generally found students with Mathematics anxiety tend to score lower in achievement tests. Mathematics anxiety is positively correlated with motivation. Students who have anxiety tend to stay away from Mathematics problems whereas those who have less anxiety stick with problems longer. The author suggested apart from creating awareness amongst teachers about the math anxiety, efforts need to be made to reduce them.

Among suggestions were, a laid-back surrounding is created. The use of a cooperative learning environment in line with social constructivist theory would aid learning. A slower learning pace is also recommended to overcome math anxiety. Game called Diff-Game was introduced and its effectiveness was tested on 117 1st year high school students in Denmark. They noted almost all students with different abilities showed improvement in their understanding for the Differentiation chapter.

Differentiation is a chapter which requires continuous and extensive practice for the mastery of skill (Pedersen et al., 2016) thus, the objective was achieved via gamification. Based on the above data on pre-test, post-test 1 and post-test 2 as well as qualitative analysis it is evident that Cartesian Pirates has positively impacted students' understanding, eliminate misconceptions and improve students' confidence in graph sketching.

Table 6: Showing the issues commonly faced by students before intervention (pre-test) and how it has been successfully addressed after intervention (post-test)

Pre-test Result	Post-test Result
<p>students cannot get the asymptote and y-intercept value of a reciprocal graph correctly.</p>  <p>no asympt no y-int.</p> <p>asymptotes mixed up.</p>	<p>Subject:</p> $y = \frac{3}{x-2} + 1 \quad x-2 \neq 0$ $x \neq 2$ <p>① $y = \frac{3}{x-2} + 1$</p> 
<p>Students cannot get domain and range of surd graph correctly.</p>  <p>domain wrong range wrong</p> <p>domain not obtained.</p>	<p>$y = -3\sqrt{1-x} + 2$</p> <p>① $y = 3\sqrt{1-x} \quad 1-x \geq 0$ $x \leq 1$</p>  <p>② $y = -3\sqrt{1-x}$</p> <p>③ $y = -3\sqrt{1-x} + 2$</p>



CONCLUSION AND RECOMMENDATION

TOWARDS STUDENTS

Gamification in graph sketching using Cartesian Pirates has brought a reshaped learning experience by creating a better engagement amongst students. The goal and engagement theory immerses the student in completing the task given. It is also believed doing activity in pairs helps the zone of proximal development of the students. Vygotsky (1978) believed that when a student is in the zone of proximal development for a particular task, providing appropriate assistance will give the student enough of a boost to achieve the task. In short, students can remember tasks better and are more careful in trying not to repeat the mistakes. Connections are much clearer now.

TOWARDS LECTURERS

For lecturers, the burden of reteaching the concept has significantly dropped. Graph sketching is a skill which is covered in Semester 1 at Matriculation College. The knowledge is vastly needed at Semester 2 and future during university courses. Previously, additional classes needed to be conducted to recap the concept. However, using Cartesian Pirates helps students to play at any time hence helping them revising the needed concepts on-the-go. Lecturers would not need to stress on misconceptions repeatedly as it is mentioned as students explore the task at their own pace. Misconception can also be monitored via Quizizz.

TOWARDS TEACHING AND LEARNING

In terms of novelty, the students would get immediate feedback and eliminate misconception during the teaching and learning process. This is important as students manage to correct their mistakes hence have a proper mental schema. Students will clear one stage before moving to the other. This is also in line with the digitalization of education which requires education to cater to the ever-demanding needs of students to understand the need of a concept. Cartesian Pirates contribute to meaningful learning as it allows students to relate known knowledge with new ones. In terms of personal learning, students can learn at their own pace outside class hours. The adaptability of the product ensures that the concept can be used across all Mathematics chapters and subjects. The next project will involve expanding the game for the Differentiation and Integration chapter. Sharing will be done with other subject lecturers to create a module that will allow students to build a proper knowledge block at any place.

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